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A new method for obtaining simple, low order models for higher order stable systems was developed and tested on a sixteenth order model of the F-100 jet engine. A scalar adaptive control procedure was extended to the multivariable case. A polynomial matrix characterization of the maximal (A,B)-invariant and controllability subspace in the kernel of C was determined together with an algorithm for the state feedback controllers which yield such maximal subspaces. This work should have significant impact in the study of systems with imprecisely known parameters. A complete resolution to the problems of determining

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state feedback invariants and canonical forms for linear systems characterized by proper rational transfer matrices was obtained. A number of results have been obtained illustrating the richness of the linkage between system theory and algebraic-geometry. For example, it has been shown that any symmetric transfer matrix over reals has a symmetric realization (answering an old question in network theory). Finally, a new, general purpose compensator for multivariable systems has been developed. This compensator insures simultaneous regulation, tracking , decoupling, stability, and robustness for a large class of linear multivariable systems.

PRACTICAL METHODS FOR THE COMPENSATION
AND CONTROL OF MULTIVARIABLE SYSTEMS

Statement of Work **AFOSR-TR- 80-0036**

The primary objective of this research effort is the development of practical methods for the compensation and control of multivariable systems. Success in this area would facilitate the design of Air Force systems as well as their control systems. Several crucial themes representing various approaches occur throughout the current study - namely -- (i) the question of approximation of dynamical behavior by simplified models - (ii) the question of parameter variation and the development of compensators which are insensitive to that variation - and (iii) the question of systems structure and qualitative properties for parameterized models and inter-connected systems. The amplification and application of a previously developed model reference technique to the question of approximation via low order models was investigated. Systems with parameters have been examined and three main lines of investigation considered namely -- (i) the relation between the geometric and polynomial matrix approaches to parameter insensitive compensation and parameterized models - (ii) the algebro-geometric method for treating multi-dimensional systems i.e. systems with transfer matrices defined over a polynomial ring in n-variables - and (iii) the use of rational mappings in studying what might be termed the algebraic linerization of certain non-linear systems. Interconnected systems were examined and a number of questions relating to decentralized spectrum assignment, stabilization, and the structure graph of an interconnected system were investigated.

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In the previous report, a new adaptive control scheme which allows arbitrary positioning of all (n) poles of an otherwise unknown, linear scalar system was described. Such a control scheme offers certain advantages in comparison to model reference adaptive controllers. Over the past year this adaptive controller has been further refined and tested, and its employment has been extended to the

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multivariable case [1]. The basic design of this adaptive controller depends on the ability to identify the unknown plant, and this requirement motivated the development of a new method for obtaining simple low order models of systems whose dynamical behavior approximates that of more complex, higher order, stable linear systems.

The problem of finding reduced order models for higher order systems, sometimes referred to as the model reduction problem, is an important one for the practicing engineer since it is difficult to apply the design procedures of modern and classical control theory to higher order systems. Numerous solutions have been proposed during the past two decades. A number of methods use time or frequency response data to directly fit low order models [2]. The procedure here falls into this category, since it determines a model which matches the frequency response of the original high order system at a certain set of pre-specified frequencies. Its primary advantage lies in the simplicity of implementation. In particular, no intermediary high order model need be calculated, only one test imput need be used, and the calculation of model parameters only requires the solution of a simple set of linear equations. The model parameters can also be obtained as the output of an analog adaptive network, since the algorithm makes use of a generalized equation error identification scheme. Finally, this algorithm readily generalizes to the multiple input-multiple output (multivariable) case where classical frequency and time domain procedures become cumbersome to apply. It should be noted that the procedure has recently been employed to derive a relatively low (fifth) order linear model of the F100 jet engine which represents its higher (sixteenth) order model behavior with very good accuracy.

Additional research under the grant has focused on understanding and applying the methods of algebraic geometry in the context of three key questions, namely:

Question 1--Can the orbits in the space of linear systems under equivalence via the action of an algebraic group be described and classified?

Question 2--What spectral structures can be achieved through the use of compensation?

Question 3--What are the essential algebraic elements required in extending results to domains other than the real and complex numbers?

A summary of various results and problems is given in [3].

More specifically, results pertaining to Question 1 were obtained in [4] for the action of state feedback and have recently been extended to systems over Noetherian integral domains of characteristic zero using a lemma of Falb to be described in the sequel. If $T(s) = R(s)P^{-1}(s)$ is a proper transfer matrix, then $(R(s),P(s))$ may be viewed as "homogeneous coordinates" of a point under right multipication via the unimodular group U_m. It was shown that the orbits under state feedback are represented by the action of stabilizer subgroups U_∂ of U_m on fibers where $\partial = \{\partial_1, \ \ , \partial_m\}$ were the Kronecker indices. The subgroup U_∂ had the structure of a semi-direct product of a normal subgroup U_N (the unipotent radical) and a reductive subgroup U_G which acts on U_N via inner automorphisms. This structure of U_∂ was a critical algebraic element in the development.

Consider now Question 2. Let A, B, C be $n \times n$, $n \times m$, $p \times n$ matrices with entries in a field k and let $\psi_{A,B}$, $\phi_{A,B,C}$ be the maps of $M_{m \times n}(k)$, $M_{m \times p}(k)$ into k^n given by

$$\psi_{A,B}(F) = \left(\mathrm{tr}(A + BF), \ldots, \frac{\mathrm{tr}(A + BF)^q}{q}, \ldots, \frac{\mathrm{tr}(A + BF)^n}{n}\right) \tag{1}$$

$$\phi_{A,B,C}(K) = \left(\mathrm{tr}(A + BKC), \ldots, \frac{\mathrm{tr}(A + BKC)^q}{q}, \ldots, \frac{\mathrm{tr}(A + BKC)^n}{n}\right) \tag{2}$$

respectively. These might be called the state feedback and output feedback trace assignment maps. Both maps are morphisms and will be almost surjective (onto) if and only if they are dominant morphisms. This leads to the following propositions:

Proposition 1 $\psi_{A,B}$ is a dominant morphism if and only if rank $\psi'_{A,B} = n$.

Proposition 2 $\phi_{A,B,C}$ is a dominant morphism if and only if rank $\phi'_{A,B,C} = n$.

Proposition 3

$$\psi'_{A,B} = \begin{pmatrix} B' \\ \vdots \\ B'(A + BF)'^{n-1} \end{pmatrix}$$

so that rank $\psi'_{A,B}$ = rank $\mathcal{C}(A,B)$ where $\mathcal{C}(A,B)$ is the controllability matrix of (A,B) .

Proposition 4

$$\phi'_{A,B,C} = \begin{pmatrix} B'C' \\ \vdots \\ B'(A + BKC)'^{n-1}C' \end{pmatrix}$$

More interesting are the implications of this approach when parameters are involved.

Consider, then, the following example: let $X = k^{2(2+1)} - Y$ with coordinates $x = (a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2)$ and $Y = \{x | (a_{21}b_1 + a_{22}b_2)b_1 - (a_{11}b_1 + a_{12}b_2)b_2 = 0\}$. Let $W \subset X$ be the variety given by $a_{11} - 1 = 0$, $a_{21} = 0$, $a_{22} - 1 = 0$, $b_2 - 1 = 0$ so that W is 2-dimensional. Let $a_{12} = p$ be a parameter. Then $\psi_{A,B}(F)$ for (A,B) in W is given by

$$\psi_{A,B}(F) = \left(2 + b_1f_1 + f_2 \ , \ 1 + pf_1 + b_1f_1f_2 + (b_1f_1 + f_2) + \frac{b_1^2f_1^2 + f_2^2}{2}\right) \tag{3}$$

and $\psi'_{A,B}$ for (A,B) in W is given by

$$\psi'_{A,B} = \begin{bmatrix} b_1 & 1 \\ p + b_1 + b_1(b_1f_1 + f_2) & 1 + b_1f_1 + f_2 \end{bmatrix} \tag{4}$$

and

$$\det\psi'_{A,B} = -p \tag{5}$$

In view of (5), it is not possible to place all poles via state feedback independently of P. However, since rank $\partial\psi'_{A,B}/\partial p = 1$ independent of (f_1,f_2) it is possible to place one (of the two) poles via state feedback independently of p. Now let $b_1 = q$ be a parameter. Since the rank of

$$\frac{\partial\psi'_{A,B}}{\partial q} = \begin{bmatrix} 1 & 0 \\ 1 + 2qf_1 + f_2 & f_1 \end{bmatrix}$$

is independent of f_2, it is possible to place one (of the two) poles via state feedback independently of q.

The observations of the example can be extended quite a bit. In fact, the following theorem has been established:

Theorem 1 Let $A(p)$, $B(p)$ depend algebraically on the parameter p and suppose that $A(p)$, $B(p)$ controllable for (almost) all p. If

$$\operatorname{rank} \frac{\partial\psi'_{A,B}}{\partial p} = \rho \tag{6}$$

is independent of $\sigma \leq n$ entries of F, then it is possible to assign σ poles independently of p.

This generalized the work of Eldem ([5]). Current work is devoted to analyzing a similar problem for output feedback.

Consider now Question 3. A detailed and extensive theory of linear systems involving transfer matrices whose entries are rational functions in one variable has been developed. Increasing interest in image processing and in the interconnection of digital and continuous systems and in systems with fixed delays has led to various attempts to extend the theory to systems involving transfer matrices whose entries are rational functions in n-variables or lie in the quotient field of an integral domain. The ultimate goal is to obtain a theory for transfer matrices whose entries lie in the quotient field of a unique factorization domain (e.g. $k[x_1, \ldots, x_n]$, k a field, $x_1, \ldots, x_n$ indeterminates). Some progress has been made and in particular, the following lemma has been established by Falb:

Lemma 1 Let R be a Noetherian integral domain of characteristic zero and let Z be the ring of integers. Then the realization, coprime factorization and trace assignment problems are solvable over $R[x_1, \ldots, x_n]$ if and only if they are solvable over $\mathbb{Z}[x_1, x_2, \ldots x_n]$.

This lemma exposes the essential algebraic nature of certain basic system theory problems and indicates the importance of studying these problems over the integers. The approach that will be used involves treatment of the entire set of linear systems. More precisely, let R be a Noetherian integral domain of characteristic zero and let $S = R[A_{ij}, B_{k\ell}, C_{rs}]$ where $i = 1, \ldots, n$, $j = 1, \ldots, n$, $k = 1, \ldots, n$, $\ell = 1, \ldots, m$, $r = 1, \ldots, p$, $s = 1, \ldots, n$. For ease of exposition, set $A = (A_{ij})$, $B = (B_{k\ell})$, $C = (C_{rs})$ and $\mathcal{C}(A,B) = [B\ AB \ldots A^{n-1} B]$, $\mathcal{O}(A,C) = [C'\ A'C' \ . \ . \ A'^{n-1} C']$ Let α,β be selections of nxn submatrices of $\mathcal{C}(A,B)$ and $\mathcal{O}(A,C)$, respectively, and set $f_\alpha = \det \mathcal{C}(A,B)_\alpha$, $g_\beta = \det \mathcal{O}(A,C)_\beta$. Then $X = \cup X_{f\alpha} \cap X_{g\beta}$ is open in Spec (S). If $Q \in GL(n,R)$, then $Q \cdot p(A,B,C) = p(QAQ^{-1}, QB, CQ^{-1})$ for $p(\cdot)$ in S defines an action and $Q \cdot p$ is prime if p is prime. Moreover, $Q \cdot X = X$. X is called the set of minimal linear systems over R. The transfer matrix can be defined for elements of X in the following way: Let K be the quotient field of R and let x be an indeterminate over S. Then

$$T(x) = \frac{C\ \mathrm{adj}(xI - A)B}{\det(xI - A)} \tag{7}$$

has entries in L(x) where L is the quotient field of S and defines a rational map of $A^{n(n+m+p)}(K) \times A^1(K) \to M_{pxm}(K)$. Since $M_{pxm}(K)$ can be naturally imbedded in a Grassmannian i.e. a projective variety π and since the fundamental set has codimension 2 in $A^1(K)$, T(x), for fixed (A,B,C) in $A^{n(n+m+p)}(K)$ extends to a rational map of $P^1(K) \to \pi$. This map is the transfer matrix and has (generic) homogeneous coordinates (N(x), D(x)). The map will be regular if and only if N(x) and D(x) are coprime. This provides a general, purely algebraic definition for coprime factorization. Additional results along these lines have been established ([6]).

In view of the increasing use of the ideas and methods of algebraic geometry in system theory, Falb has begun work on a book on the subject. The book has several aims, namely: (i) to provide, in a motivated context, an account of the algebraic geometry needed in system theory; and (ii) to develop the system theory results which are of a purely algebraic nature. The main focus will be on the three key questions raised earlier. The first chapter has been written and is topically outlined below.

Chapter I

A. Algebraic Methods

B. Notion of System

1. Representations
2. Equivalence of Representations
3. Space of Minimal Linear Systems

C. Qualitative Properties

1. Compensation
2. Pole-Assignment and Stability
3. Parameter Variation

Frequency domain methods have always dominated control system design in the scalar (single input/output) case, when compared to the more "modern" state-space or differential operator methods, due to the relative simplicity of the resulting controllers and their ability to function acceptably over a rather wide range of plant parameter variations; i.e. their robustness. It is not surprising, therefore, that numerous studies have been made to "extend" various frequency domain techniques to the multivariable case in order to simply and reliably achieve a diversity of desired design goals. In most cases, however, direct extensions of scalar frequency domain procedures, such as the Nyquist stability criteria or the root locus, are not possible and often rather complex modifications have had to be made to existing theories in order to achieve appropriate design objectives. Further complicating the picture is the fact that noninteration (or decoupling) is often an addditional design objective in the multivariable case, and a completely decoupled, stable system cannot always be achieved by the relatively simple feed-forward controllers obtained by multivariable, frequency domain methods. Nevertheless, many of the designs which have been developed using multivariable frequency domain methods do perform quite well in practice and have therefore found rather widespread acceptance in a variety of applications.

On the other hand, the so called "modern" methods, which have generally relied on exact knowledge of the plant, are continually being improved upon and extended to take into consideration parameter uncertainty and/or variations; i.e. robustness is becoming increasingly important in designs based on state-space or differential oporator methods. Although these "modern" methods generally imply more complex controller configurations, than those associated with frequency domain methods, they are less heuristic to implement and can generally achieve more than is possible with the simpler controllers designed by frequency domain methods. Moreover, with the ever increasing utilization of computers in the control loop, it may be argued that controller simplicity is no longer as important as it once was, and one might therefore expect to see more complex controllers being used in future applications.

In light of these observations, recent research has outlined a new procedure for designing controllers which simultaneously achieve a variety of desired design goals in deterministic, unity feedback, linear multivariable system [7]. More specifically, a new algorithm has been obtained for the systematic design of a "three part" multivariable controller which simultaneously insures

(a) a non interactive or decoupled closed loop design,

(b) complete and arbitrary closed loop pole placement, which implies desired (single loop) transient performance as well as closed loop stability,

(c) zero steady-state errors between the plant outputs and any nondecreasing deterministic inputs,

(d) complete steady-state output rejection of nondecreasing deterministic disturbances, and

(e) robustness with respect to stability, disturbance rejection, and zero error tracking for rather substantial plant parameter variations.

The development employs the more "modern" (Laplace transformed) differential operator approach for controller synthesis, which involves transfer matrix factorizations and the manipulation of polynomial matrices in the Laplace operator s.

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Personnel

1. William A. Wolovich (co-principal investigator)
2. Peter L. Falb (co-principal investigator)
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Presentations

[A] H. Elliott and W. A. Wolovich "Parameter Adaptive Identification and Control" IEEE CDC San Diego, CA Jan. 10-12, 1979.

[B] W. A. Wolovich and Pedro Ferreira "Output Regulation and Tracking in Linear Multivariable Systems" IEEE CDC San Diego, CA Jan 10-12, 1979.

[C] W. A. Wolovich Linear Multivariable Systems 2 week seminar presented at the Ecole Nationale Superieure de Mecanique, Universite de Nantes, France June 18-29, 1979.

[D] P. L. Falb "Application of Algebraic Geometry and System Theory" NATO Conference on Algebraic Methods in System Theory Cambridge, MA June, 1979.

[E] P. L. Falb "Generic Properties of Systems Defined by Transfer Matrices" MIT Lab for Information and Decision Sciences Sept. 20, 1979.

[F] H. Elliott and W. A. Wolovich "Parameter Adaptive Control of Linear Multivariable Systems" 13th Annual Asilomar Conference 5-7 November, 1979.